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# Level 4

# Focused Mathematics Intervention

**Teacher's Guide** 

Teacher Created Materials

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# **Kit Components**

#### HOW TO USE This product

#### **Teacher's Guide**

30 easy-to-use, standards-based lesson plans



#### **Student Guided Practice Book**

Full-color student activities



#### **Assessment Guide**

Includes a pretest, posttest, performance tasks with assessments, and the answer key for the *Student Guided Practice Book* 



#### 3 Math Fluency Game Sets

Include a game board, directions, an answer key, and game pieces



#### 3 Digital Math Fluency Games

Focus on mathematical skills and strategies, and are on the Digital Resources USB Device



#### **Digital Resources**

- PDFs of all student materials, game sets, activity sheets, assessments, etc.
- PDFs of teacher resources
- Digital Math Fluency Games
- Electronic versions of the Pretest, Posttest, Performance Tasks, and reporting tools

#### Refocus Mini Lesson



Provide as PowerPoint<sup>®</sup> and PDF files



# Teaching a Lesson

#### Teacher's Guide

Each 8-page lesson is organized in a consistent format for ease of use. Teachers may choose to complete some or all of the lesson activities to best meet the needs of their students. Lesson materials can be utilized flexibly in a variety of settings. For example, modeling with a small group, using printed materials with a document camera, or using PDF materials on a digital platform, such as an interactive whiteboard. Each lesson includes:

- an overview page with key information for planning
- key mathematics content standards covered
- key mathematical practices and processes addressed
- an overview providing teacher background or student misconceptions
- a Warm-Up activity to build students' recall of important mathematical concepts
- a whole-class Language and Vocabulary activity
- time markers to indicate the approximate time for instruction
- a whole-class section focusing on the key concept/skill being taught
- use of the gradual release of responsibility model in the Whole-Group lesson section
- differentiation strategies to support and extend learning with the Refocus lesson and Extend Learning activity
- math fluency games that motivate students to develop and reinforce mastery of basic skills
- a Math in the Real World concept task activity









HOW TO USE

THIS PRODUCT

# Teaching a Lesson (cont.)

#### Student Guided Practice Book

HOW TO USE

THIS PRODUCT

Each lesson in the *Teacher's Guide* has seven corresponding student pages in the *Student Guided Practice Book:* 

- a We Do activity to support the gradual release of responsibility model
- a You Do activity to facilitate independent practice
- a Quick Check to easily monitor students' progress
- a Refocus activity for students who need more instruction
- an Independent Practice page to reinforce mathematical content taught in the lesson
- a Math in the Real World concept task for students to apply the math concept in a real-life scenario
- a Reflection page for students to share their mathematical understanding



# Learning Objectives

#### Numbers and Operations—Fractions

• Compare two fractions with different numerators and different denominators by creating common denominators. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions; e.g., by using a visual fraction model.

#### **Mathematical Practices and Processes**

- Reason abstractly and quantitatively.
- Model with mathematics.
- Look for and express regularity in repeated reasoning.

# **Progress Monitoring**

The *Student Guided Practice Book* pages below can be used to formally and informally assess student understanding of the concepts.

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# Materials

• Student Guided Practice Book (pages 132–138)

**ESS**M

- Math Fluency Game Sets
- Digital Math Fluency Games
- chart paper
- markers
- index cards
- sentence strips
- yellow and blue crayons or colored pencils

### Teacher Background

To successfully compare fractions, students need to have a strong understanding of the meaning of the numerator and denominator in a fraction. They must recognize that the denominator is the number of equal parts the whole is divided into and the numerator is the number of equal parts being represented. To create fractions with common denominators, students should also be comfortable with generating equivalent fractions (see Lesson 18).

# Warm-Up 10 min.

LESSON

- 1. Put students into groups of two or three. Provide each group with a sheet of chart paper.
- 2. Explain that each group will choose a fraction and write it at the top of their chart paper. Then, they will represent the fraction using a method of their choice. For example, students could plot the fraction on a number line, create an area model, or represent parts of a group.



**3.** Allow groups several minutes to work. Then, bring the class back together. Have groups share their representations. Ask students to explain their representations. For example: Why did you divide the number line this way? How did you know how many parts on the model to shade?

### Language and Vocabulary 10 min.

1. Write the following vocabulary terms on the board:

compare	numerator	denominator	fraction
greater than	less than	equal	

- **2.** Create two sets of index cards. One set should contain each word. The other set should contain the definition of each word.
- **3.** Put students into seven groups. Give each group a definition card. Say, "Today, we are going to play I Have, Who Has. I have a set of index cards with each of the vocabulary words on it. I will pick a card and say, 'I have...' and read the word. Then, I will say, 'Who has the definition?' You will look at your definition and see if you think it matches my word card. If you think your group has the definition, raise your hands."
- **4.** Play I Have, Who Has with students. Choose one word card at a time, and have students identify who has the definition.



#### Focus

- The following lesson will address this focus question: What strategies can you use to compare fractions?<sup>2</sup>
- **2.** You may wish to write the focus question on the board and read it aloud to students. Explain that you will revisit the focus question at the end of the lesson.

#### (I Do)

. Say, "Today, we are going to compare fractions." Write  $\frac{2}{5}$  and  $\frac{4}{5}$  on the board. Say, "Let's compare these fractions using an area model. How could we do this?" Students should identify that they can draw two equal rectangles, divide them into five equal parts, and then shade two parts for  $\frac{2}{5}$  and four parts for  $\frac{4}{5}$ . Draw area models to represent  $\frac{2}{5}$  and  $\frac{4}{5}$  on the board.



- **2.** Ask, "How do we know which fraction is greater?" Students should recognize that the fraction with more of the area shaded is greater  $(\frac{4}{5})$ . Review comparison symbols with students (>, <, and =), and write an inequality to compare  $\frac{2}{5}$  and  $\frac{4}{5}$  (e.g.,  $\frac{2}{5} < \frac{4}{5}; \frac{4}{5} > \frac{2}{5}$ ). Remind students that they can only compare fractions when they refer to the same whole. For example, each of the area models is the same size, so the comparison is valid.
- **3.** Ask, "What do you notice about the size of the parts in these fraction models?" Students should recognize that the size of the parts is the same because the denominator is the same. Ask, "Which fraction represents more equal parts?"  $(\frac{4}{5})$  Say, "It makes sense that  $\frac{4}{5}$  is greater. In  $\frac{4}{5}$ , we have four equal parts. In  $\frac{2}{5}$ , we have only two equal parts. Let's see if we can find this pattern when we compare more fractions with the same denominator.

#### Language Support

Every time you write an inequality, have students read it aloud with you to reinforce the vocabulary and meaning of the symbols. LESSON

#### Whole-Group Lesson (cont.)

- **4.** Repeat Steps 1–2 to compare the following fractions:  $\frac{5}{6}$  and  $\frac{2}{6}$   $(\frac{5}{6} > \frac{2}{6})$ ;  $\frac{3}{10}$  and  $\frac{8}{10}$   $(\frac{3}{10} < \frac{8}{10})$ ; and  $\frac{5}{12}$  and  $\frac{10}{12}$   $(\frac{5}{12} < \frac{10}{12})$ . By the last set of fractions, encourage students to reason about the fractions without the visual model. They should recognize that  $\frac{10}{12}$  is greater than  $\frac{5}{12}$  because the equal parts are the same size, and there are more parts represented in  $\frac{10}{12}$  than in  $\frac{5}{12}$ .
- **5.** Say, "This method works when we have fractions with the same denominator. But, what if the fractions did not have the same denominator?" Write  $\frac{2}{4}$  and  $\frac{3}{8}$  on the board. Ask students to identify how they could compare these fractions. Students may suggest creating area models. Create area models on the board.



- 6. Say, "Making a model is a good strategy. But what if we could make the fractions have the same denominators? Then, we could just compare the numerators. Is there a way that we could make these fractions share a common denominator?" Students should recognize that they can rewrite  $\frac{2}{4}$  as an equivalent fraction with a denominator of 8. Ask, "How can we do this?" Work together with students to multiply the numerator and denominator by 2 to generate the equivalent fraction  $\frac{4}{8}$ . Ask, "How can we compare the fractions now?" Students should recognize that now that the denominators are the same, they can simply compare the numerators. Four is greater than three, so  $\frac{4}{8}$  is greater than  $\frac{3}{8}$ . Record  $\frac{2}{4} > \frac{3}{8}$  on the board.
- 7. Repeat Step 6 to compare fractions by making common denominators:
  - $\frac{2}{6}$  and  $\frac{2}{3} \left( \frac{2 \times 2}{3 \times 2} = \frac{4}{6}; \frac{2}{6} \text{ is less than } \frac{4}{6}; \frac{2}{6} < \frac{2}{3} \right)$
  - $\frac{4}{5}$  and  $\frac{6}{10} \left( \frac{4 \times 2}{5 \times 2} = \frac{8}{10}; \frac{8}{10} \text{ is greater than } \frac{6}{10}; \frac{4}{5} > \frac{6}{10} \right)$
  - $\frac{1}{2}$  and  $\frac{4}{12} \left( \frac{1 \times 6}{2 \times 6} = \frac{6}{12}; \frac{6}{12} \text{ is greater than } \frac{4}{12}; \frac{1}{2} > \frac{4}{12} \right)$

LESSON

I Do

(cont.)



### Whole-Group Lesson (cont.)

- We Do
- 1. Refer students to the Compare Fractions activity sheet (*Student Guided Practice Book*, page 132). Say, "Let's compare more fractions." Copy Question 1 onto the board:  $\frac{5}{8} \boxed{\phantom{3}8}$ .
- 2. Ask, "How can we compare these fractions?" Students should recognize that the fractions have the same denominator. Allow them to draw a model to visually compare the fractions, if needed. If they can simply reason about the numerators, they can also explain their thinking in words (e.g., *The denominators are the same, so I know the parts are the same size. Because*  $\frac{5}{8}$  *is greater, it has more parts than*  $\frac{3}{8}$ ). Ensure that students record the correct symbol to compare the fractions. ( $\frac{5}{8} > \frac{3}{8}$ )
- **3.** Copy Question 2 onto the board:  $\frac{4}{6}$   $\boxed{\frac{4}{12}}$ . Ask, "How can we compare these fractions?" Students should recognize that they can draw area models or make common denominators. Ask, "How can we make common denominators?" Students should identify that  $\frac{4}{6}$  can be multiplied by  $\frac{2}{2}$  to make  $\frac{8}{12}$ . Allow them to show these steps on the activity sheet as you do so on the board. Then, they should compare the numerators. Students should conclude that  $\frac{8}{12}$  is greater than  $\frac{4}{12}$ , so therefore  $\frac{4}{6}$  is greater than  $\frac{4}{12}$ .  $(\frac{4}{6} > \frac{4}{12})$
- 4. Use these steps to solve Questions 3–4 with students. Then, direct students to Question 5. Ask, "How can we find out if Maria or Sadie ate more sandwich?" Students should recognize that they need to compare the fractions. Allow them to solve with a partner and share their solutions with the class. Students should have created common denominators  $(\frac{1 \times 4}{2 \times 4} = \frac{4}{8})$  and compared  $\frac{4}{8}$  and  $\frac{5}{8}$ , determining that  $\frac{5}{8}$  is greater. This means that Maria ate more. Make sure students understand that the comparison is only valid because the sandwiches are the same size.
- **5.** Students will complete the activity sheet by explaining how they solved. To help students explain their reasoning, provide them with the following sentence frames:
  - To compare the fractions, I \_\_\_\_\_
  - \_\_\_\_\_ is greater than/less than \_\_\_\_\_. So \_\_\_\_\_ ate more sandwich.

### Whole-Group Lesson (cont.)

#### You Do

LESSON

- 1. Refer students to the Compare Some More activity sheet (*Student Guided Practice Book*, page 133). Provide the sentence frames from Step 5 of the We Do section to help students explain how they solved.
- **2.** Have students share their solutions. If students have difficulty explaining their reasoning, remind them to use the sentence frames and vocabulary terms.

#### Closing the Whole-Group Lesson

Revisit the focus question for the lesson: *What strategies can you use to compare fractions?* Students should identify strategies used in the lesson. Make sure to recognize when certain strategies should be used. For example, when students have two fractions with the same denominator, they can model them with area models, or reason about the number of parts in the fractions (the numerators). When the denominators are the same, they know that the size of the parts is the same, so the fraction with more parts (i.e., the greater numerator) is greater. When students have two fractions with different denominators, they can model these with area models and compare. Or they can try to rewrite the fractions to have a common denominator. Then, they can compare the numerators directly to see which is greater or less. Make sure students recognize that they can only compare numerators in this way when the fractions have the same denominators.

# Progress Monitoring (5) min.

- 1. Have students complete the Quick Check activity sheet (*Student Guided Practice Book*, page 134) to gauge student progress toward mastery of the Learning Objectives. Provide students with scratch paper to show their work on the selected response questions.
- 2. Based on the results of the Quick Check activity sheet and your observations during the lesson, identify students who may benefit from additional instruction in the Learning Objectives. These students will be placed into a small group for reteaching. See instructions on the following page.



# Differentiated Instruction 20 min.

Gather students for reteaching. The remaining students will complete the Independent Practice activity sheet (*Student Guided Practice Book*, page 136) to reinforce their learning and then play the Math Fluency Games.

#### Refocus

Revisit the focus question for the lesson: What strategies can you use to compare fractions? Provide students with sentence strips. Write the following on the board: Compare  $\frac{3}{4}$  and  $\frac{1}{2}$ . Have students use sentence strips to model each fraction. Ask guiding questions (e.g., How many equal parts should you fold the paper into? How many equal parts should you shade?) Then, have students compare the models to determine which fraction is greater.  $(\frac{3}{4})$  Show students how to fold the  $\frac{1}{2}$  strip in half to create fourths. Ask students what fraction is shown now.  $(\frac{2}{4})$  Write  $\frac{2}{4}$  on the board, and ask students how they could compare the fractions. They should conclude that because both models are in fourths, it is easier to see that  $\frac{3}{4}$  is greater than  $\frac{2}{4}$ . Repeat with the fractions  $\frac{5}{8}$  and  $\frac{3}{4}$ .  $(\frac{3}{4} \text{ is greater})$  Then, fold the  $\frac{3}{4}$  model in half to see the equivalent fraction.  $(\frac{6}{8})$ 

Support students as they complete Question 1 on the Refocus activity sheet (*Student Guided Practice Book*, page 135). Have them solve Questions 2–4 independently or with a partner. Students will draw visual models to compare two fractions, and then divide one of the models to create models with common denominators.

#### Math Fluency Games



Math Fluency Game Sets



**Digital Math Fluency Games** 

# Extend Learning

Have students complete the Lesson 19 Extend Learning Task (filename: extendtask19.pdf). Students will write a compare word problem given two fractions. They will trade papers with a partner and solve each other's word problems. Ensure that students write problems in which the whole is the same size in both fractions.

# Math in the Real World 30 min.

- 1. Refer students to the Math in the Real World: Pizza Leftovers task (*Student Guided Practice Book*, page 137). Have a student read the task aloud. Tell students to explain or summarize the task to their partner. Have a few students share their summaries.
- 2. Ask students to think about what information they will need to solve the task and what the task is asking them to do. Then, have them share with a partner. Ask a few students to share aloud. Students should identify that they know the pizzas are the same size and they know how many slices each pizza was cut into. They also know how many slices of each pizza were eaten. Students need to find out which pizza had more eaten. Have students work in groups of two or three to complete the task.
- 3. As students are working, circulate and ask focusing, assessing, and advancing questions:
  - What fraction can you use to show the amount of the cheese/veggie pizza that was eaten?
  - How can you compare the fractions?
  - Why is it important that the pizzas are the same size?

#### Sentence Frames for Explaining Reasoning

- The fraction of the cheese/veggie pizza eaten is \_\_\_\_\_
- I can compare the fractions by \_\_\_\_\_.
- \_\_\_\_\_ is the larger fraction. This means \_\_\_\_\_.
- **4.** Observe how students are solving the task, and choose a few groups who solved the task in different ways to share their solutions and reasoning. Try to have the solutions move from concrete representations to more abstract representations. For example, have students share solutions using visual representations (creating area models to compare the fractions), then more abstract strategies such as creating a common denominator and comparing the numerators. Make sure students explain their reasoning as they share solutions.
- **5.** As groups are sharing their solution paths, reasoning, and strategies, ask questions:
  - Do you agree or disagree with the solution path and reasoning? Why?
  - Which solution path makes the most sense to you? Why?

# Lesson Reflection (5) min.

Have students summarize their learning about comparing fractions, and provide feedback on any questions they still have about the content on the Reflection activity sheet (*Student Guided Practice Book*, page 138).

LESSON

Name:\_\_\_\_\_

LESSON

Date: \_\_\_\_\_

# **Compare Fractions**

**Directions:** Compare the fractions. Use >, <, or =. Show your work or explain your thinking in words.



#### Directions: Solve.

**5** Sadie ate  $\frac{1}{2}$  of a sandwich. Maria ate  $\frac{5}{8}$  of a sandwich that is the same size. Who ate more sandwich?

#### Solution: \_\_\_\_\_

S Explain how you solved.



# **Compare Some More**

**Directions:** Compare the fractions. Use >, <, or =. Show your work or explain your thinking in words.



\_\_\_\_\_

#### Directions: Solve.

3 Les is baking a pie. He needs to add  $\frac{1}{2}$  of a cup of butter. He will also add  $\frac{10}{12}$  of a cup of flour. Does the pie need more butter or flour?

Solution: \_\_\_\_\_

4 Mrs. Perez walks on the track with her class. On Monday, the class walked  $\frac{3}{4}$  of a mile. On Tuesday, they walked  $\frac{7}{12}$  of a mile. Did they walk farther on Monday or Tuesday?

Solution: \_\_\_\_\_

Choose Question 3 or 4. Explain how you solved.

Name:\_\_\_\_\_

LESSON

Date:





#### Directions: Solve.

**5** Aubrie worked on homework for  $\frac{4}{5}$  of an hour. Her brother worked for  $\frac{9}{10}$  of an hour. Who worked on homework for less time?

\_\_\_\_\_

Solution: \_\_\_\_\_

Explain how you solved.





**Directions:** Model each fraction. Show how you can divide one of the models to make common denominators. Then, compare the fractions.



Name:

\_\_\_\_\_

LESSON

Date: \_\_\_\_\_

# **Independent Practice**

**Directions:** Compare each fraction. If the comparison is true, color the square blue. If the comparison is false, color the square yellow.



#### Name:\_

Date:



Joy bought pizza to share with her friends. She ordered a cheese pizza and a veggie pizza. They were the same size. The cheese pizza was cut into 6 slices. The friends ate 5 of the slices. The veggie pizza was cut into 12 slices. They ate 9 of the slices. Which pizza did they eat more of?



LESSON

Name:\_\_\_\_\_

LESSON

Date: \_\_\_\_\_

# Reflection 1 Why do you need to have the same size whole when comparing fractions? 2 What is a question that you still have about comparing fractions?

	Pretest			
1.	<ul> <li>Which of the following describes this equation: 4 x 9 = 36?</li> <li>A Nine times as many as four is 36.</li> <li>Nine more than four is 36.</li> <li>Thirty-six is nine times fewer than four.</li> <li>Nine times as many as 36 is four.</li> </ul>	3. V s t t r ( ( ( (	What equation can you use to solve this problem? A fishing pole costs \$30. That is three times more than a fishing net. How much does the fishing net cost? (a) $30 \ge 3 = \$90$ (b) $5 \ge 6 = \$30$ (c) $30 \div 3 = \$10$ (d) $30 \div 10 = \$4$	
2.	<ul> <li>Which situation can be modeled by the equation 8 x 7 = 56?</li> <li>A Jan has 8 dolls. Sarah has 7.</li> <li>Jan has 8 dolls. Sarah has 7 times fewer dolls than Jan.</li> <li>Sarah has 8 dolls. Jan has 7 dolls more than Sarah.</li> <li>Jan has 8 dolls. Sarah has 7 times as many dolls as Jan.</li> </ul>	4. 7 a b c c r a a ( ( ( ( (	<ul> <li>The fourth graders are going on an all-day field trip. They need 120 bottles of water. James prought two cases of 24 bottles each and Ming brought four cases of 12 bottles each. How many more bottles of water are needed?</li> <li>A 24 bottles</li> <li>B 48 bottles</li> <li>C 14 bottles</li> <li>D 96 bottles</li> </ul>	

Go On

#### Performance Task 1 Our New Building

#### Part A

Mrs. Robinson's math class is helping her draw a map of the school's new building. The building contains three halls, a cafeteria, an office, and a gym. Mrs. Robinson has prepared a diagram of the building:



- 1. There is a Hall 3 that is the same size and shape as Hall 1 and Hall 2. Draw Hall 3 on the diagram, to the right of Hall 2.
- 2. How many pairs of parallel lines are there in Hall 3?
- **3.** What is the name of the shape formed by each of the halls? How do you know this is the correct name?
- **4.** What is the name of the angles formed by the walls in the cafeteria? How do you know this is the correct name?